2024/FYUG/EVEN/SEM/
STADSC–151T/066

FYUG Even Semester Exam., 2024

STATISTICS

(2nd Semester)

Course No. : STADSC–151T

(Probability Distribution)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—A

Answer any *ten* of the following questions : 2×10=20

1. Define discrete random variable with examples. State two properties of random variable.

2. If F is the distribution function of a random variable X and if $a < b$, then show that

$$P(a < X \leq b) = F(b) - F(A)$$

3. Explain conditional probability distribution of Y under the condition that $X = x$ in both discrete and continuous case.

4. Show that for a random variable X, $E(X^2) \geq \{E(X)\}^2$.

5. Let X be a random variable with probability density function

$$f(x) = \begin{cases} 2x; & 0 < x < 1 \\ 0; & \text{elsewhere} \end{cases}$$

Find $E(X)$ and $P\left(-\frac{1}{2} < X < \frac{1}{2}\right)$.

6. Define mathematical expectation of discrete and continuous random variable. Prove that

$$E(aX + b) = aE(X) + b$$

where a and b are constants.

7. Define moment-generating function and characteristic function for both discrete and continuous random variables.

8. State the properties of characteristic function.

9. Prove that the moment-generating function of the sum of two independent random variables is equal to the product of their moment-generating functions.

10. X is a binomial variate with mean 4 and variance 2. Find $P(3 < X \leq 5)$.

11. What is hypergeometric distribution?

12. Define geometric distribution and obtain its mean.

13. Define beta distribution of second kind and find its mean.

14. Write the definition of continuous uniform distribution and obtain its mean.

15. Discuss at least two importances of normal distribution in Statistics.

SECTION—B

Answer any *five* of the following questions : 10×5=50

16. (a) Define probability density function (p.d.f.). Let the p.d.f. of a random variable X is given by

$$f(x) = 6x(1 - x)\,; \quad 0 \leq x \leq 1$$

Determine the value of the constant b such that $P(X < b) = P(X > b)$. 1+3=4

(b) Define cumulative distribution function. What is the lower limit of cumulative distribution function? A random variable X has the following probability function :

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find K, $P(0 \le X < 4)$, $P(2 \le X \le 5)$, $P(X \le 5)$ and $P(1 < X \le 6)$. 6

17. (a) Define joint probability density function. Prove that if X and Y are independent continuous random variables, then the p.d.f. of $U = X + Y$ is

$$h(u) = \int_{-\infty}^{\infty} f_X(V) f_Y(U - V)\, dV$$ 1+3=4

(b) Define joint distribution function of two-dimensional random variable. Two discrete random variables X and Y have

$$P(X = 0, Y = 0) = \frac{2}{9}$$

$$P(X = 0, Y = 1) = \frac{1}{9}$$

$$P(X = 1, Y = 0) = \frac{1}{9}$$

$$P(X = 1, Y = 1) = \frac{5}{9}$$

Find the marginal distribution X and Y; conditional probability distribution of X given $Y = 1$. Examine whether X and Y are independent. 2+1+1=4

(c) Let joint p.d.f. of (X, Y) is

$$f(x, y) = Axy \; ; \quad 1 \le x \le y,\ 1 \le y \le 2$$
$$= 0 \; ; \quad \text{Otherwise}$$

Find the value of A. Hence obtain the marginal distribution of X and Y. 1+1=2

18. (a) Prove that mathematical expectation of the sum of two random variables is equal to the sum of their individual expectations, provided all the expectations exist. 3

(b) The bivariate probability distribution of two random variables X and Y is given below :

Y \ X	−1	0	1
−1	0	0·1	0·1
0	0·2	0·2	0·2
1	0	0·1	0·1

Find $E(X)$, $E(Y)$, $V(X)$ and $V(Y)$. 4

(c) A continuous random variable X has the following p.d.f. :

$$f(x) = x/2 \quad ; \quad 0 \le x \le 1$$
$$= 1/2 \quad ; \quad 1 < x \le 2$$
$$= (3-x)/2 \; ; \quad 2 < x \le 3$$

Find $E(X)$ and $E(X^2)$. 3

19. (a) State and prove the multiplication theorem of expectation. 3

(b) If the possible values of a random variable X are 0, 1, 2, ⋯, then show that

$$E(X) = \sum_{n=0}^{\infty} P(X > n)$$ 3

(c) Let X and Y have the joint probability density function

$$f(x, y) = \begin{cases} 2; & 0 < x < y < 1 \\ 0; & \text{otherwise} \end{cases}$$

Show that the conditional mean and variance of X given $Y = y$ are $y/2$ and $y^2/12$ respectively. 4

20. (a) Define cumulant-generating function of a random variable. Discuss the effect of change of origin and scale on cumulants. 1+3=4

(b) State the uniqueness theorem of moment-generating function. Let the random variable X assume the value r with probability law

$$P(X = r) = q^{r-1}p; \quad r = 1, 2, 3, \cdots$$

Find the moment-generating function and hence obtain mean and variance. 1+3=4

(c) Show that the characteristic function of a random variable is uniformly continuous. 2

21. (a) Obtain mean, variance, μ_3 and μ_4 of a random variable in terms of cumulants. 3

(b) If $M(t)$ is the moment-generating function of a random variable X about origin, show that the moment μ_r' is given by

$$\mu_r' = \left[\frac{d^r}{dt^r} M(t)\right]_{t=0}$$ 2

(c) Define conditional expectation and conditional variance of discrete and continuous random variable X given $Y = y$ and Y given $X = x$. Also prove that

$$E(X) = E[E(X \mid Y)]$$ 2+3=5

22. *(a)* Obtain mean and variance of binomial distribution with parameters n and p. 3

(b) If X and Y are independent Poisson variates, then show that the conditional distribution of X given $X+Y$ is binomial. 3

(c) Find the mean of discrete uniform distribution. Explain how you will use hypergeometric distribution to estimate the number of fish in a pond. 1+3=4

23. *(a)* Define negative binomial distribution. Obtain the moment-generating function (m.g.f.) and hence obtain mean and variance. 1+2+1+2=6

(b) Give some examples of occurrence of Poisson distribution in different fields. Obtain the recurrence relation between moments of Poisson distribution

$$\mu_{r+1} = \lambda\left(r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right)$$

where μ_r is the rth moment about mean λ. 1+3=4

24. *(a)* Show that for a normal distribution with mean μ and variance σ^2

$$\mu_{2n} = 1.3.5.\cdots(2n-1)\sigma^{2n}$$ 4

(b) Let X have a standard Cauchy distribution. Find the p.d.f. of X^2 and identify its distribution. 3

(c) Obtain the mean and the variance of beta distribution of first kind. 3

25. *(a)* Obtain the moment-generating function of normal distribution with mean μ and standard deviation σ. Prove that a linear combination of independent normal variates is also a normal variate. 2+3=5

(b) Define Laplace distribution and write its characteristic function. 2

(c) Write the p.d.f. gamma distribution and obtain its m.g.f. 3

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